Direction of Arrival Estimation (DOA) in Interference & Multipath Propagation

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Scope

• There are many location methods
  - **Source location** vs. **Self-location** (Navigation)
  - **Active** vs. **Passive**
  - **Network based (GPS)** vs. **Single platform** (DOA or AOA)

• We will concentrate on source location with a single platform equipped with a sensor array, passive

• Emphasis on DOA estimation

• Narrowband signals only
Outline

• Basics of DOA Estimation
   Beamforming Type
   High Resolution Type

• Adaptive Beamforming DOA Estimation in Strong Interference

• High Resolution DOA Estimation in Multipath
  • Smoothing Method

• Source Association and Locating the Sources
Outline

• Basics of DOA Estimation
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  ▪ Smoothing Methods

• Source Association and Locating the Sources
Objective of DOA Estimation

- To find DOA (relative to the array orientation) of all incident RF rays
- Could have multipath propagation and interference
Narrowband Signal Sources

• A complex sinusoid

\[ s(t) = \alpha e^{j\beta} e^{j\omega t} = \rho e^{j\omega t} \]

• A real sinusoid is a sum of two sinusoids

\[ \alpha \cos(\omega t + \beta) = \frac{\alpha}{2} e^{j\beta} e^{j\omega t} + \frac{\alpha}{2} e^{-j\beta} e^{-j\omega t} = \rho_1 e^{j\omega t} + \rho_2 e^{-j\omega t} \]

• A delay of a sinusoid is a phase shift

\[ s(t - t_0) = e^{-j\omega t_0} \rho e^{j\omega t} = e^{-j\omega t_0} s(t) \]

• Apply approximately to narrowband signals
A Uniform Linear Array

A signal source \( s(t) = \rho e^{j\omega t} \) “impinges” on the array with an angle \( \theta_0 \)

\( c \): propagation speed

- If the received signal at sensor 1 is \( x_1(t) = s(t) \)
- Then it is delayed at sensor \( i \) by \( \Delta_i = \frac{(i-1)d \sin \theta_0}{c} \)
- Then the received signal at sensor \( i \) is

\[
x_i(t) = e^{-j\omega \Delta_i} x_1(t) = e^{-j\omega \Delta_i} s(t) = e^{-j\omega \frac{(i-1)d \sin \theta_0}{c}} s(t)
\]
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Beamforming

- Delay-and-sum type
- Reverse the delay/phase on each sensor to “line-up” the received signal phase
- Adding all phase-shifted sensor outputs to enhance the received SNR in that direction
Beamforming

• By adjusting the delay or phase shifts, we electronically steer the beam through all look directions to find DOA of an incident signal.

• Received signal at the $i$th sensor:

$$x_i(t) = e^{-j\omega \Delta_i} x_1(t) = e^{-j\omega \Delta_i} s(t) = e^{-j\omega \frac{(i-1)d \sin \theta_0}{c}} s(t)$$

• $\theta_0$ is the incidence angle of the received signal.

• The delay or phase shift of the beamformer on the $i$th sensor is computed according to a “look angle” $\theta$ to get the output:

$$y(t) = \sum_{i=1}^{N} w_i^* x_i(t) = \sum_{i=1}^{N} e^{j\omega \frac{(i-1)d \sin \theta}{c}} x_i(t)$$

$$= \sum_{i=1}^{N} e^{j\omega \frac{(i-1)d \sin \theta - \sin \theta_0}{c}} s(t)$$
Beamforming

• If $\theta = \theta_0$, then the output becomes

$$y(t)\bigg|_{\theta=\theta_0} = \sum_{i=1}^{N} e^{j\omega(c(i-1)d[\sin \theta - \sin \theta_0])} s(t)\bigg|_{\theta=\theta_0} = \sum_{i=1}^{N} s(t) = Ns(t)$$

• I.e., the output is enhanced $N$ times
• Since noise is not correlated, noise is not enhanced
• So SNR is enhanced $N$ times
• At other steering angles, the complex weights may cancel so the output is not enhanced or even degraded
• This is the designed purpose of a beamformer
Beamforming

- Beam pattern: A gain pattern as a function of the steering angle

\[ G(\theta, \theta_0) = \left| \sum_{i=1}^{N} e^{j\omega (i-1)d \left[ \sin \theta - \sin \theta_0 \right]} \right| \]

\[ \theta_0 = 0 \]

\[ \theta_0 = 30^\circ \]
Beamforming

- Sensor spacing is usually wavelength/2
- Larger spacing creates “grating lobes” that confuse with the main lobe
- Smaller spacing reduces the total aperture – lower spatial resolution
Beamforming

- Number of sensors (array aperture) are directly related to spatial resolution

4-element (aperture = 1.5 wavelength)  7-element (aperture = 3 wavelength)
Beamforming

- Problem 1: A wide mainlobe causes poor spatial resolution

  Two targets 10 degrees apart

  Cannot distinguish them
Beamforming

- Problem 2: A strong interferer can come into a sidelobe and completely swamp weak signal in the look direction.

Control sidelobes by Dolph-Chebyshev shading, w. limited effect.
Pros/Cons of Beamforming

- Can find only one DOA at a time
- Spatial resolution determined by number of sensors in an array, but generally not very good unless having a large number of sensors
- Works for other array shapes also, need to know sensor positions in an array
- Sensitive to sensor position, gain, and phase errors, must calibrate carefully to make it work well
- Interference is a big problem
- Multipath is a much lesser problem
Outline

• Basics of DOA Estimation
  ▪ Beamforming Type
  ▪ High Resolution Type

• Adaptive Beamforming DOA Estimation in Strong Interference

• High Resolution DOA Estimation in Multipath
  ▪ Smoothing Method

• Source Association and Locating the Sources
MUSIC

• Able to find DOAs of multiple sources in “one-shot”
• High spatial resolution compared with beamforming methods (I.e., a few antennas can result in very high spatial resolution)
• MUSIC stands for **Multiple Signal Classifier**
Narrowband Signal Sources

• Consider $I$ narrowband signal sources

\[ s_1(t) = \rho_1 e^{j\omega_1 t}, \quad s_2(t) = \rho_2 e^{j\omega_2 t}, \quad \ldots, \quad s_I(t) = \rho_I e^{j\omega_I t} \]

• Assume that all amplitudes are uncorrelated

\[ E\{\rho_i \rho_j\} = \begin{cases} \sigma_i^2; & i = j \\ 0; & i \neq j \end{cases} \]

• Recall received signal on the $i$th sensor:

\[ x_i(t) = e^{-j\omega \Delta_i} x_1(t) = e^{-j\omega \Delta_i} s(t) = e^{-j\omega (i-1)d \sin \theta_0/c} s(t) \]
Signal Model

- Put received signals at all \( N \) sensors together:

\[
x(t) = \begin{bmatrix}
    x_1(t) \\
    x_2(t) \\
    x_3(t) \\
    \vdots \\
    x_N(t)
\end{bmatrix} = \begin{bmatrix}
    1 \\
    -j\omega \frac{d\sin \theta}{c} \\
    e^{-j\omega \frac{2d\sin \theta}{c}} \\
    \vdots \\
    e^{-j\omega \frac{(N-1)d\sin \theta}{c}}
\end{bmatrix} \begin{bmatrix}
    s(t) = a(\theta)s(t)
\end{bmatrix}
\]

- \( a(\theta) \) is called a “steering vector”
Signal Model

If there are \( I \) source signals received by the array, we get a “signal model”:

\[
x(t) = As(t) + n(t)
\]

- \( x(t) \) --- received signal vector (\( N \) by 1)
- \( s(t) \) --- source signal vector (\( I \) by 1)
- \( n(t) \) --- noise vector (\( N \) by 1)
- \( A = [a(\theta_1), \cdots, a(\theta_I)] \) (\( N \) by \( I \))
- \( s(t) = [s_1(t), \cdots, s_I(t)]^T \)

- Sources are independent, noises are uncorrelated
- Column of \( A \) can also be normalized
The MUSIC Algorithm

• Compute the $N \times N$ correlation matrix

\[ \mathbf{R}_x = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma_0^2 \mathbf{I} \]

where \( \mathbf{R}_s = E\{\mathbf{s}(t)\mathbf{s}^H(t)\} = \text{diag.}\{\sigma_1^2, \ldots, \sigma_I^2\} \)

• If the sources are somewhat correlated so \( \mathbf{R}_s \) is not diagonal, it will still work if \( \mathbf{R}_s \) has full rank.

• If the sources are correlated such that \( \mathbf{R}_s \) is rank deficient, then it is a problem. A common solution is “spatial smoothing”.

Q: Why is the rank of \( \mathbf{R}_s \) (being \( I \)) so important?

A: It defines the dimension of the signal subspace.
The MUSIC Algorithm

• For $N > I$, the matrix $\mathbf{A}\mathbf{R}_s\mathbf{A}^H$ is singular, i.e.,

\[
\det[\mathbf{A}\mathbf{R}_s\mathbf{A}^H] = \det[\mathbf{R}_x - \sigma_0^2 \mathbf{I}] = 0
\]

• But this implies that $\sigma_0^2$ is an eigenvalue of $\mathbf{R}_x$

• Since the dimension of the null space of $\mathbf{A}\mathbf{R}_s\mathbf{A}^H$ is $N-I$, there are $N-I$ such eigenvalues $\sigma_0^2$ of $\mathbf{R}_x$

• Since $\mathbf{R}_x$ is non-negative definite, there are $I$ other eigenvalues $\sigma_i^2$ such that $\sigma_i^2 > \sigma_0^2 > 0$

• Let $\mathbf{u}_i$ be the $i$th eigenvector of $\mathbf{R}_x$ corresponding to $\sigma_i^2$

\[
\mathbf{R}_x \mathbf{u}_i = \sigma_i^2 \mathbf{u}_i; \quad i = 1, 2, \ldots, N
\]

\[
\sigma_i^2 > \sigma_0^2, \ i = 1, \ldots, I; \quad \sigma_i^2 = \sigma_0^2, \ i = I + 1, \ldots, N
\]
The MUSIC Algorithm

\[ R_x u_i = [\text{AR}_s A^H + \sigma_0^2 I] u_i = \sigma_i^2 u_i; \quad i = 1, 2, \ldots, N \]

- This implies

\[ \text{AR}_s A^H u_i = (\sigma_i^2 - \sigma_0^2) u_i; \quad i = 1, 2, \ldots, N \]

\[ \text{AR}_s A^H u_i = \begin{cases} (\sigma_i^2 - \sigma_0^2) u_i; & i = 1, 2, \ldots, I \\ 0; & i = I + 1, \ldots, N \end{cases} \]

- Partition the \( N \)-dimensional vector space into the signal subspace \( U_s \) and the noise subspace \( U_n \)

\[
\begin{bmatrix} U_s & U_n \end{bmatrix} = \begin{bmatrix} u_1 & \cdots & u_I & u_{I+1} & \cdots & u_N \end{bmatrix}
\]

\[
U_s: \sigma_i^2, \ i \leq I \quad U_n: \sigma_i^2 = \sigma_0^2, \ i > I
\]
The MUSIC Algorithm

- The steering vector $a(\theta_i)$ is in the signal subspace
- Signal subspace is orthogonal to noise subspace

$$AR_s A^H u_i = \begin{cases} \left(\sigma_i^2 - \sigma_0^2\right) u_i; & i = 1, 2, \ldots, I \\ 0; & i = I + 1, \ldots, N \end{cases} \quad (1)$$

(1) means $I$ linear combinations of columns of $A$ equal the signal subspace spanned by columns of $U_s$

(2) means the linear combinations of columns of $A$, i.e., the signal subspace, is orthogonal to $U_n$
The MUSIC Algorithm

- The steering vector $\mathbf{a}(\theta_i)$ is in the signal subspace.
- Signal subspace is orthogonal to noise subspace.
- This implies that $\mathbf{a}^H(\theta_i)\mathbf{U}_n = 0$.
- So the MUSIC algorithm searches through all angles $\theta$, and plots the “spatial spectrum”

$$P(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{U}_n}$$

- Wherever $\theta = \theta_i$, $P(\theta)$ exhibits a peak.
- Peak detection will give spatial angles of all incident sources.
The MUSIC Algorithm

1. Compute the signal correlation matrix $R_x$
2. Perform SVD/EVD on $R_x$ and separate the smallest eigenvalues from larger eigenvalues
3. Eigenvectors corresponding to the smallest eigenvalues form a noise subspace $U_n$
4. Search through all angles $\theta$ in the MUSIC spatial spectrum
   \[ P(\theta) = \frac{1}{a^H(\theta)U_n} \]
5. Peaks correspond to DOAs
The MUSIC Algorithm

MUSIC spatial spectrum compared with other methods
Pros/Cons of The MUSIC Algorithm

- Can find multiple DOAs with high resolution
- Number of sensors must be more than number of sources
- Works for other array shapes also, need to know sensor positions in an array
- Very sensitive to sensor position, gain, and phase errors, need careful calibration to make it work well
- Searching through all $\theta$ could be computationally expensive
- Interference is not much a problem, just another source whose DOA can be found with other sources
- Multipath is a big problem
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• Basics of DOA Estimation
  ▪ Beamforming Type
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  ▪ Smoothing Methods

• Source Association and Locating the Source
Adaptive Beamforming

• Rather than fixed weights/phase, adapt the weights according to the signal environment
• The generalized sidelobe canceller (GSC) achieves this w. flexible constraints
• Delay-and-sum beamformer output $y(t)$:
  \[ y(t) = a^H(\theta)x(t) = a^H(\theta)a(\theta_0)s(t) \]

• The GSC output $y(t)$:
  \[ y(t) = w^Hx(t) = w^Ha(\theta_0)s(t) \]

$w$ is a vector of complex weights, more general
Adaptive Beamforming

Q: How do we choose the weights $w$?

A: By some constrained optimization

Constraints:

1) Array gain at the look direction should be preserved

\[ y(t) = w^H x(t) = w^H a(\theta_0) s(t) \]

\[ w^H a(\theta_1) = a^H (\theta_1) w = 1 \]

When $\theta_1 = \theta_0$, we have $y(t) = s(t)$

2) Array gain at some other directions may need to be zero

\[ w^H a(\theta_2) = a^H (\theta_2) w = 0 \]

This is also called null-steering
Adaptive Beamforming

• Jointly, we write these constraints as

\[
\begin{bmatrix}
a^H(\theta_1) \\
a^H(\theta_2)
\end{bmatrix} w = [a(\theta_1), a(\theta_2)]^H w = C^H w = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = g
\]

• More constraints, other constraints, can be imposed

• The cost function is:

\[
E\{ |y(t)|^2 \} = w^H E\{x(t)x^H(t)\}w = w^H R_x w
\]

• The optimization problem is:

\[
\min_w w^H R_x w \quad \text{subject to} \quad C^H w = g
\]
Adaptive Beamforming

• The solution to this optimization problem is:

\[ w = R_x^{-1}C(C^H R_x^{-1}C)^{-1}g \]

• Since we minimize the output energy, a strong interferer will result in a deep notch in its direction.
Adaptive Beamforming

• Another interpretation: Decompose $w$ into

$$w = w_G - Bw_I$$

$B$ is the orthogonal compliment of $C$:

$$C^H B = 0$$

• Upper branch: Main channel; Lower branch: Auxiliary channel
Adaptive Beamforming

\[ w_G = C(C^H C)^{-1} g \] so that \[ C^H w_G = g \]

- I.e., \( w_G \) allows gain (null) in the desired directions – main channel
- Since \( C^H B = 0 \), the matrix \( B \) is a “blocking matrix” that blocks the signal (null) in the desired directions – auxiliary channel
- \( w_I \) is designed to produce a replica of interference leaking into the main channel to subtract it out

\[ w_I = (B^H R_x B)^{-1} B^H R_x w_G \]
Adaptive Beamforming

- Advantages of adaptive beamforming
  - Able to reduce effect of interferences from unknown angles
  - Can steer look direction and multiple null directions
  - Get a signal copy easily

- Shortcomings of adaptive beamforming
  - Spatial resolution is still low
  - Need enough weights (antenna channels) to obtain enough degrees of freedom: At least # of constraints + 1
  - Sidelobe levels may not be controlled perfectly
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Multipath Propagation Issue

- Urban areas have a lot of multipaths
- Beamforming DOA algorithms are not affected by this much
- MUSIC fails in multipath!
- This is because if there is multipath, two or more DOAs will be from the same source – i.e., some sources in MUSIC signal model are correlated
- Now with $I$ DOAs there are less than $I$ sources
Multipath Propagation Issue

• Correlated sources disable MUSIC!

• E.g.: \( s(t) = [s_1(t), as_1(t)]^T \)

\[
R_s = E\{s(t)s^H(t)\} = \begin{bmatrix}
\sigma_1^2 & a\sigma_1^2 \\
a\sigma_1^2 & a^2\sigma_1^2
\end{bmatrix}
\]

• This is called rank deficiency
  – The rank of \( AR_sA^H \) will be less than \( I \)
  – The signal subspace has dimension less than \( I \)
  – There are less than \( I \) peaks in the MUSIC spectrum
  – Which of the \( I \) DOAs will give less than \( I \) peaks?

• In fact all peaks will be wrong.
Multipath Propagation Issue

• Remedy: Spatial Smoothing, still find $I$ DOAs
  – $L$ overlapping subarrays
  – $M$ sensors in each subarray, $M > I$
  – $N$ total sensors
  – $N = M + L - 1$

\[
\begin{array}{cccccccc}
1 & 2 & \ldots & L & \ldots & M & M+1 & N \\
\end{array}
\]

• Let $\mathbf{x}_i(t)$ be the received signal vector of the $i$th subarray
Multipath Propagation Issue

- One signal case:

\[
x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_N(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -j\omega \frac{d\sin \theta}{c} \\ e \\ -j\omega \frac{2d\sin \theta}{c} \\ \vdots \\ e^{-(N-1)d\sin \theta} \end{bmatrix} s(t) = a(\theta) s(t)
\]

\[x_1(t) = a_M(\theta) s(t), \quad x_2(t) = e^{-j\omega \frac{d\sin \theta}{c}} a_M(\theta) s(t), \text{ etc.}\]

- In general \[x_i(t) = e^{-j\omega \frac{(i-1)d\sin \theta}{c}} a_M(\theta) s(t)\]
Multipath Propagation Issue

• With \( I \) DOAs:

\[
x_i(t) = e^{-j\omega(i-1)d\sin\theta_i/c} a_M(\theta_1) s_1(t) + \cdots + e^{-j\omega(i-1)d\sin\theta_i/c} a_M(\theta_I) s_I(t)
\]

\[
= \begin{bmatrix} a_M(\theta_1) & \cdots & a_M(\theta_I) \end{bmatrix} A_M
\]

\[
= \begin{bmatrix} e^{-j\omega(i-1)d\sin\theta_i/c} & \cdots & O \\ O & \cdots & e^{-j\omega(i-1)d\sin\theta_i/c} \end{bmatrix}
\]

\[
\Rightarrow x_i(t) = A_M D_i s(t) \quad \text{(Noise has been ignored so far)}
\]
Multipath Propagation Issue

• Compute correlation matrix of each subarray

\[ R_{x_i} = E\{x_i(t)x_i^H(t)\} = A_M D_i R_s D_i^H A_M^H + \sigma_0^2 I \]

• Now average the correlation matrices of all subarrays

\[ R_{x_L} = \frac{1}{L} \sum_{i=1}^{L} E\{x_i(t)x_i^H(t)\} = A_M \left[ \frac{1}{L} \sum_{i=1}^{L} D_i R_s D_i^H \right] A_M^H + \sigma_0^2 I \]

• Note the dimension of \( A_M \) is \( M \) by \( I \)

• The dimension of the matrix in the brackets is \( I \) by \( I \)
Multipath Propagation Issue

$$R_{xL} = \frac{1}{L} \sum_{i=1}^{L} E\{x_i(t)x_i^H(t)\} = A_M \left[ \frac{1}{L} \sum_{i=1}^{L} D_i R_s D_i^H \right] A_M^H + \sigma_0^2 I$$

- The matrix in the brackets has full rank $I$ if $L$ is large enough, i.e., $L \geq I$
- Provided $M > I$, there is a noise subspace in $R_{xL}$
- Now can apply MUSIC to $R_{xL}$
- Will detect $I$ DOAs with this spatial smoothing
- Price paid is that more sensors are required: $M > I$, $L \geq I$

$M + L = N \quad \rightarrow \quad L$ more sensors to “de-correlate”
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Source Association Under Multipath

• Several paths belong to one source, multiple sources
• Can detect all DOAs of multiple paths and multiple sources
• But how many sources are there?
• Need to know which DOAs can be associated to which sources – source association
• Then locate the sources in several ways
Source Association Under Multipath

- Once sources are associated with DOAs
  - Locate the sources by using multipath if we know the reflection geometry – back tracing

- Multipath works to our advantage in this case
Source Association Under Multipath

• Alg. works for any DOA method, easier with MUSIC

1. Compute the received signal correlation matrix based on all sensors, as in MUSIC but un-smoothed

2. Compute the noise subspace matrix $\mathbf{U}_n$ whose rank is the number of sources $J$, not the number of paths $I$

3. Based on the estimated DOAs, compute the overall steering matrix $\mathbf{A}$

4. Find minimum eigenvalues of $\mathbf{A}^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{A}$ and their corresponding eigenvectors $\mathbf{q}_i$, the number of these eigenvalues is the number of sources $J$

5. Construct $I$ by $J$ matrix $\mathbf{Q} = [\mathbf{q}_1 \ldots \mathbf{q}_J]$
Source Association Under Multipath

6. Find $J$ groups of rows of $Q = [q_1 \cdots q_J]$ that are independent among the groups but dependent within each group, these correspond to groups of paths associated with each source.

- Dependency test can be done by, e.g., dividing the corresponding elements in two rows and compute the variance of the ratios.
Source Association Under Multipath

- E.g.: 15 antennas in a ULA, two sources, 5 paths
  - DOA from Source 1: $-30^\circ$ and $-60^\circ$
  - DOA from Source 2: $-5^\circ$, 25$^\circ$, and 55$^\circ$
  - We only know 5 DOAs rank ordered as:
    $-60^\circ$, $-30^\circ$, $-5^\circ$, 25$^\circ$, 55$^\circ$

- Put them into $A$ in this order
- Computed eigenvalues of $A^H U_n U_n^H A$ which are
  \{20.0559, 17.9585, 14.8226, 0.0430, 0.0088\}
- Obviously $I = 5$ but $J = 2$
Source Association Under Multipath

\[
Q = [q_1 \quad q_2] = \\
\begin{bmatrix}
0.4152 + j0.4824 & 0.2656 + j0.1121 \\
0.4062 + j0.4989 & 0.2913 + j0.1122 \\
-0.2290 + j0.0746 & 0.3954 - j0.3430 \\
-0.2429 + j0.0498 & 0.3883 - j0.3502 \\
-0.2210 + j0.1123 & 0.2770 - j0.4418
\end{bmatrix}
\]

- Dependency test on the rows of \( Q \)
  1 & 2 belong to one
  3, 4, 5 belong to the other
Source Association Under Multipath

• When reflection geometry is unknown
  – Identify the LOS paths from the reflected paths if either the source or the receiver is moving

• Once LOS paths are identified, the sources can be more easily located by
  – Ray back tracing
  – Several different angles of LOS DOA to triangulate
Source Association Under Multipath

- Stationary receiver: All paths have fixed DOAs;
- Moving receiver: Paths have varying DOAs, DOA variations differ between LOS path and reflected paths.
Source Association Under Multipath

Most likely, reflected paths appear and disappear intermittently as one travels, but not the LOS paths.

All DOAs need to be tracked.

Reflected paths may be eliminated over time by choosing the longest continuous path as the LOS path.
Source Association Under Multipath

- Caution: If not tracked properly, two DOAs may be erroneously identified after they cross
- Special measures are needed to prevent this
Conclusions

• Beamforming (adaptive)
  – Can reduce effect of interferences from unknown DOAs
  – Can steer look direction and multiple null directions
  – Spatial resolution is low
  – Resilient to multipath propagation

• MUSIC (with smoothing)
  – Can find multiple DOAs with high resolution
  – Very sensitive to sensor position, gain, and phase errors, need careful calibration to make it work well
  – Spatial smoothing is difficult to achieve on other than ULA

• Source association can be applied to both methods
• Source location done by ray back trace/triangulation
About GIRD Systems, Inc.

- Founded in 2000 - based in Cincinnati
- Specializes in communications and signal processing, especially developing novel algorithms to solve challenging problems
- As of Jan. 2010, won more than 15 Phase I awards and 7 Phase II awards from Navy, Air Force, Army
- Partnerships with many large contractors including Northrop Grumman, L-3 Communications, etc.
About GIRD Systems, Inc.

• Key Technology Areas
  – Interference Mitigation (no reference, in-band)
  – Direction Finding (wideband, high-resolution)
  – Location/Navigation (Assisted GPS, GPS denied, signals of opportunity)
  – Wireless Network Security (physical layer)
  – Power Amplifier Linearization
  – Novel Communications systems/modeling

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